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Can Ignorance Promote Democracy?

Jevin D. West, *et al.*

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non, the founder of information theory. Shannon defined the entropy of a single random variable, and laid the groundwork for what we now call the mutual information, MI, of a pair of random variables. This quantity turns out to be a new measure of dependence and was first proposed as such in 1957 (*10*). Reshef *et al.*'s MIC is the culmination of more than 50 years of development of MI.

What took so long, and wherein lies the novelty of MIC? There were three difficulties holding back MI's acceptance as the right generalization of the correlation coefficient. One was computational. It turns out to be surprisingly tricky to estimate MI well from modest amounts of data, mainly because of the need to carry out two-dimensional smoothing and to calculate logarithms of proportions. Second, unlike the correlation coefficient, MI does not automatically come with a standard numerical range or a ready interpretation of its values. A value of $r = 0.5$ tells us something about the nature of a cloud of points, but a value of MI = 2.2 does not. The formula $[1 - \exp(-2\mathrm{MI})]^{1/2}$ in (*10*) satisfies all the requirements for a good measure of dependence, apart from ease of computation, and ranges from 0 to 1 as we go from independence to total dependence. But Reshef *et al.* wanted more, and this takes us to the heart of MIC. Although r was introduced to quantify the association between two variables evident in a scatter plot, it later came to play an important secondary role as a measure of how tightly or loosely the data are spread around the regression line(s). More generally, the coefficient of determination of a set of data relative to an estimated curve is the square of the correlation between the data points and their corresponding fitted values read from the curve. In this context, Reshef *et al.* want their measure of association to satisfy the criterion of equitability, that is, to assign similar values to "equally noisy relationships of different types." MI alone will not satisfy this requirement, but the three-step algorithm leading to MIC does.

Is this the end of the Galton-Pearson correlation coefficient r? Not quite. A very important extension of the linear correlation r_{XY} between a pair of variables X and Y is the partial (linear) correlation $r_{XY.Z}$ between X and Y while a third variable, Z, is held at some value. In the linear world, the magnitude of $r_{XY.Z}$ does not depend on the value at which Z is held; in the nonlinear world, it may, and that could be very interesting. Thus, we need extensions of MIC(X,Y) to MIC$(X,Y|Z)$. We will want to know how much data are needed to get stable estimates of MIC, how susceptible it is to outliers, what three- or higher-dimensional relationships it will miss, and more. MIC is a great step forward, but there are many more steps to take.

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PSYCHOLOGY

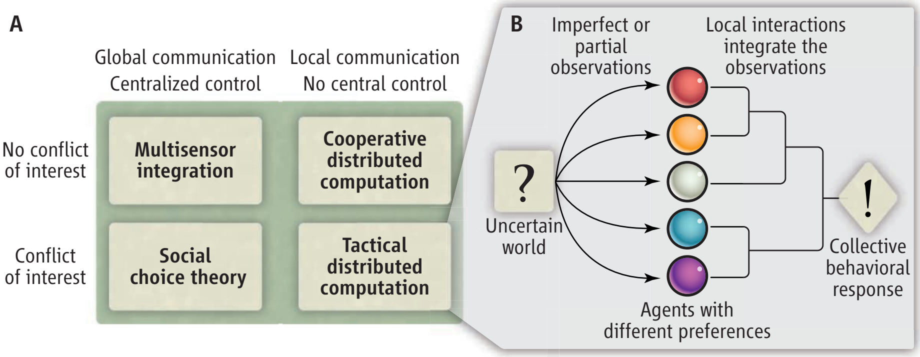
Can Ignorance Promote Democracy?

When a group needs to reach a consensus decision, uninformed members can help to reduce the influence of a manipulative minority.

Jevin D. West[1] and Carl T. Bergstrom[1,2]

Ideas are like fire, observed Thomas Jefferson in 1813—information can be passed on without relinquishing it (*1*). Indeed, the ease and benefit of sharing information select for individuals to aggregate into groups, driving the buildup of complexity in the biological world (*2*, *3*). Once the members of some collective—whether cells of a fruit fly or citizens of a democratic society—have accumulated information, they must integrate that information and make decisions based upon it. When these members share a common interest, as do the stomata on the surface of a plant leaf (*4*), integrating distributed information may be a computational challenge. But when individuals do not have entirely coincident interests, strategic problems arise. Members of animal herds, for example, face a tension between aggregating information for the benefit of the herd as a whole, and avoiding manipulation by self-interested individuals in the herd. Which collective decision procedures are robust to manipulation by selfish players (*5*)? On page 1578 of this issue, Couzin *et al.* (*6*) show how the presence of uninformed agents can promote democratic outcomes in collective decision problems.

[1]Department of Biology, University of Washington, Box 351800, Seattle, WA 98195–1800, USA. [2]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA. E-mail: jevinw@u.washington.edu, cbergst@u.washington.edu



Distributed information processing. (**A**) Research in this domain comprises four areas: multisensor integration (*12*), social choice theory (*13*), cooperative distributed computation (*14*), and tactical distributed computation (*5*). The Couzin *et al.* study lies in the lower right quadrant, where the challenges of both social choice and distributed computation must be solved. (**B**) In this schematic of tactical distributed computation, an uncertain world is observed imperfectly by agents with different preferences. By means of local interactions, they aggregate the information and preferences to arrive at a collective decision.

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Research in distributed information processing broadly falls into four domains (see the figure, panel A), which differ depending on whether there is local or central control over the collective decision and on whether the agents share common interests (*7*). The scenario addressed by Couzin *et al*. lies in the particularly challenging lower right quadrant. In this domain, one must simultaneously consider both the local nature of information exchange in distributed systems and the strategic issues that arise in social choice theory.

We can think of tactical distributed control as having two stages (see the figure, panel B). In an initial social choice stage, each agent imperfectly observes the world and selects a preferred outcome. In a subsequent distributed computation stage, individual preferences are aggregated through local interactions among agents to select a consensus decision. In such a situation, agents can pursue selfish aims not only through strategic choice of preferred outcome (*8*)—much as a far-left voter might back a moderate democrat with a chance of winning instead of a fringe candidate with a more liberal platform— but also through tactical behavior during the process of local information exchange.

Couzin *et al*. consider cases in which the group must decide between two options. Allowing only two options simplifies the problem considerably: There is no incentive for strategic voting, but incentives remain for manipulating the process of information integration. Furthermore, when groups must select among more than two options, they face a host of voting paradoxes. Thomas Jefferson's acquaintance and correspondent Marquis de Condorcet noted the basic reason for this more than two centuries ago (*9*): If a group has intransitive preferences—its members collectively prefer A to B and B to C in pairwise comparisons, yet they also prefer C to A—there is no straightforward way to select a single best course of action. This poses a serious social choice problem, because even when no single individual has intransitive preferences, the aggregate preferences of the group can be intransitive.

With two options, the task of determining the majority opinion is analogous to the classic density classifier problem in the study of distributed computation (*10*). But whereas the vast majority of work on the density classifier looks at cooperative distributed computation, Couzin *et al*. look at—and even implement in a vertebrate system, namely schooling fish—an extension to the noncooperative case.

The authors develop three different models of the information integration process. In each of these, agents are probabilistically influenced to adopt the opinions of their neighbors, and can promote their own opinions by being reluctant to change them. In this way, an intransigent minority can convert the entire group over to their minority opinion. Such behavior can impose costs on the group, including a reduced responsiveness to the state of the environment, an increased time to make a collective decision, and an increased risk of group fragmentation.

One might expect groups with uninformed members to be particularly susceptible to tactical behavior by minority subpopulations. If that tactical behavior involved some sort of active proselytizing to accelerate conversion to the minority opinion, one would be right. But Couzin *et al*. show that when the tactical behavior involves intransigence, uninformed individuals have the opposite effect. Their presence allows the majority to wrest control back from a manipulative minority. In each of their models, this occurs because the uninformed individuals tend to adopt the opinions of those around them, amplifying the majority opinion and preventing erosion by an intransigent minority. In this way, adding uninformed individuals to a group can facilitate fair representation during the process of information integration. Jefferson's passionate arguments on the importance of education for democratic society notwithstanding (*11*), Couzin *et al*. have identified circumstances in which ignorance can promote democracy.

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GEOCHEMISTRY

A World Awash with Nitrogen

James J. Elser

Human disruption of the nitrogen cycle has left signs across the Northern Hemisphere since about 1895 C.E.

For most of the history of the biosphere, nitrogen has swirled tantalizingly out of reach: In the form of inert N_2 gas (78% of the modern atmosphere), it was available only to certain bacteria and cyanobacteria capable of producing the nitrogenase enzyme that breaks the strong N–N triple bond. Even these nitrogen fixers cannot liberally fix nitrogen because of the high energy costs of running nitrogenase and the high demands for other elements needed to produce nitrogenase. It is for this reason that plant and algal biomass and productivity of many ecosystems are limited by nitrogen (*1*) and that the supply of nitrogen plays a major role in structuring plant communities (*2*). A report by Holtgrieve *et al*. on page 1545 of this issue (*3*) and other recent studies (*4*, *5*) shed light on the extent to which human activities have changed nitrogen availability, with implications for ecosystems around the world.

Humans, like all species, long operated within the constraints of the natural nitrogen cycle. In early agricultural periods, humans returned animal and human wastes to the fields, thereby adding nitrogen (and phosphorus) and enhancing crop productivity.

School of Life Sciences, Arizona State University, Tempe, AZ 85287, USA. E-mail: j.elser@asu.edu